Exercise 40

Find the derivative of the function.

$$y = e^{\sin 2x} + \sin(e^{2x})$$

Solution

Take the derivative using the chain rule.

$$\begin{aligned}
y' = \frac{dy}{dx} &= \frac{d}{dx}\left[e^{\sin 2x} + \sin(e^{2x})\right] \\[2mm]
&= \frac{d}{dx}\left(e^{\sin 2x}\right) + \frac{d}{dx}\left[\sin(e^{2x})\right] \\[2mm]
&= e^{\sin 2x} \cdot \frac{d}{dx}(\sin 2x) + \cos(e^{2x}) \cdot \frac{d}{dx}(e^{2x}) \\[2mm]
&= e^{\sin 2x} \cdot (\cos 2x) \cdot \frac{d}{dx}(2x) + \cos(e^{2x}) \cdot (e^{2x}) \cdot \frac{d}{dx}(2x) \\[2mm]
&= e^{\sin 2x} \cdot (\cos 2x) \cdot (2) + \cos(e^{2x}) \cdot (e^{2x}) \cdot (2) \\[2mm]
&= 2\left[e^{\sin 2x} \cos 2x + e^{2x} \cos(e^{2x})\right]
\end{aligned}$$